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Weapon System Selection for Capability-Based Defense Planning using Lanchester Models integrated with Fuzzy MCDM in Computer Assisted Military Experiment

Hakan Ayhan Dağıstanlı[1,*],
[1] Department of Industrial Engineering, Turkish Military Academy, National Defence University, Ankara, Turkey

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ABSTRACT

Capability-based planning (CBP) is an approach that is increasingly gaining importance in modern defense strategies. Unlike traditional threat-focused planning, capability-based planning aims to enable countries to develop flexible and sustainable military capabilities against changing security environments. In this context, weapon system selection should be evaluated not only against current threats but also in terms of suitability for possible future operational requirements. The selection of weapon systems requires consideration of a large number of technical, operational and financial criteria. In this process, Fuzzy Multi-Criteria Decision Making (FMCDM) methods are frequently used. In addition, simulation techniques are a critical tool for testing the effectiveness of weapon systems on the battlefield by creating scenarios close to reality. Lanchester equations, in particular, are an effective tool for analyzing the combat dynamics of forces. In this study, an Anti Guided Tank Missiles selection methodology in which the Fuzzy Analytic Hierarchy Process (FAHP) is integrated with Lanchester equations is applied to an example. JCATS was used for computer assisted military experiments scenario analysis. The results show that Lanchester equations are valid.

1. Introduction

In developed countries, the armed forces form a fundamental defense concept for the implementation of security policies. To efficiently fulfill such a task, it is of paramount importance for countries' security to determine and implement the optimal capability requirements, ensuring that the armed forces are prepared for the most likely threats to national security. Capability-based planning, a relatively new paradigm, provides an analytical framework for strategic or long-term planning [1].

Capability-based defense planning aims to develop flexible and adaptable military capabilities by anticipating potential threats and risks in the future. Within this approach, the selection of weapon systems is made to meet not only specific threats but also broader operational requirements. Unlike traditional threat-based planning, in capability-based defense planning, weapon systems are chosen not to counter a specific adversary, but to possess versatile capabilities

* Corresponding author.
E-mail address: hakanayhand@gmail.com

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that can adapt to the changing security environment. In this context, decision-makers create a long-term defense strategy by evaluating technological advancements, cost-effectiveness analyses, and the interoperability features of weapon systems [2].

In the weapon system selection process, factors such as threat analysis, operational needs, and the cost-benefit balance are taken into consideration [3]. Primarily, multi-role and modular systems suitable for the future combat environment are preferred to ensure flexibility. Additionally, elements such as interoperability with allied countries and ease of logistical support play a critical role in weapon system selection. Finally, considering resource allocation and sustainability factors, systems that not only meet operational requirements but also provide long-term economic and strategic benefits are prioritized [4].

Simulations play a crucial role in the weapon system selection process by testing different combat scenarios and identifying the most suitable systems [5]. Within capability-based defense planning, simulations allow different threat environments, geographical conditions, and combat dynamics to be virtually modeled. In this process, the performance, effectiveness, and adaptability of weapon systems are analyzed in detail [6]. For example, by using computer-based wargames and AI-supported simulations, the integration of air, land, and naval forces is tested, and the most suitable system combinations are determined. Thus, before transitioning to real operations, the strengths and weaknesses of systems are identified, and the most effective and cost-efficient solutions are chosen.

Simulations not only assess the performance of existing weapon systems but also help predict the impact of newly developed systems in the combat environment. For instance, by simulating the effectiveness of different air defense systems against enemy air threats, optimal engagement strategies can be developed. Similarly, the mobility, detection capabilities, and strike effectiveness of modern systems can be tested in various scenarios to optimize performance. In this way, decision-makers can scientifically select the systems that will best enhance operational success and avoid costly mistakes.

In this study, Joint Conflict and Tactical Simulation (JCATS) scenarios were used for the selection of a sample weapon system. To test the effectiveness of weapon systems, Lanchester strategies were applied. The weights of the weapon systems' attrition capabilities in the Lanchester equations were calculated using the Multi-Criteria Decision-Making method, Fuzzy Analytical Hierarchy Process (FAHP). The second section of the study presents a literature review, the third section discusses the Lanchester Laws, the fourth section details FAHP, and the fifth section provides a case study on the effectiveness of anti-tank systems against a main battle tank. The sixth section presents the results.

2. Literature Review

Capability-based defense planning is an approach aimed at developing flexible and adaptable military capabilities in uncertain and dynamic threat environments [7]. Researchers such as Correia (2019) argue that, compared to traditional threat-based planning, the capability-based approach provides greater capacity to respond effectively to unforeseen crises [8]. In this context, the evaluation of weapon systems should be considered not only in terms of their effectiveness against current threats but also with respect to their suitability for potential future operational requirements. Hodický et al. (2020) emphasize that simulations and operational analyses play a critical role in weapon system selection for various scenarios in such planning. As widely accepted in the literature, considering factors like flexibility, scalability, and interoperability in capability-based defense planning ensures the long-term effectiveness of modern weapon systems [9].

The selection of weapon systems in the defense industry should be approached not only through technological competence and cost factors but also within the framework of analytical approaches such as Multi-Criteria Decision-Making (MCDM) methods [10]. Analytical Hierarchy Process (AHP), developed by Saaty (1980), and fuzzy logic-based methods proposed by Yager (1988), highlight the importance of multi-criteria decision analysis in the defense field [11,12]. Dağdeviren et al. (2009) stress that analytical methods are indispensable for comparing different weapon systems in terms of operational effectiveness, logistical sustainability, and cost [13]. Particularly, Karaburun and Alaykıran (2018) suggest that multi-criteria optimization processes regarding the integration of weapon systems and their suitability for the modern battlefield provide significant advantages to decision-makers [14]. Studies in the literature demonstrate that the use of analytical methods leads to more efficient allocation of defense resources and minimizes subjectivity in system selection processes [15-17].

In the process of selecting and evaluating the effectiveness of weapon systems, Fuzzy Multi-Criteria Decision-Making (Fuzzy MCDM) and Lanchester equations emerge as powerful analytical tools in military planning. The fuzzy logic theory, proposed by Zadeh (1965), enables more flexible and realistic analyses in decision-making processes where uncertainties prevail [18]. Tzeng and Huang (2011) have shown that fuzzy logic-based MCDM methods make decision processes objective, particularly in military supply chains and weapon system selection [19]. On the other hand, the Lanchester equations, developed by Frederick W. Lanchester, mathematically model the dynamics of military units in conflict, analyzing the effect of force size and firepower on the outcomes of wars [20]. Researchers like Özdağoğlu (2019) emphasize the importance of updating Lanchester's equations with asymmetric warfare and technological superiority factors in modern warfare conditions for strategic foresight [21]. The literature reveals that both methods should be used as strategic decision-support tools in defense planning, and integrated analytical approaches lead to healthier decision-making processes [22-27].

3. Lanchester Laws

The course of all wars depends on the combatants involved and the nature of the battlefields. This has been mathematically validated [28]. Despite the long time since their introduction, the Lanchester Laws of Combat remain valid today. They have evolved based on the comparison between preferring a larger fleet or a smaller but more advanced one. Over time, the method has also tried to respond to the question of predicting casualty numbers before entering a war.

Frederick William Lanchester (1868-1946), a British engineer, was one of the first to use mathematical models to analyze military operations during World War I. His work led to the development of a model showing how opposing forces would attrit each other in a conflict. Lanchester proposed two types of combat models. The first law is often referred to as the law of linearity, while the second law is known as the law of squares, or the modern warfare law. Many studies have found solutions using these laws [29-33]. The Lanchester laws have certain assumptions.

a) The war continues until one of the units is destroyed.
b) Units do not receive support during the war.
c) Units destroy each other at a constant rate.
d) All weapons are used effectively, with no errors considered.
e) Units do not use maneuvers or other tactics that provide combat superiority.
f) Units are homogeneous in structure.

3.1. Linear Law

This model was developed by considering ancient warfare, which involved more direct one-on-one combat and lacked equipment to provide a significant advantage in war. Lanchester proposed a simple but reasonable condition for this type of war. In these types of wars, combatants armed with swords and spears can only attack a single target at a time within their range. Therefore, the winning side in such wars must be proportional to the number of combatants engaged in direct physical contact with the enemy. The laws have been explained with examples in various sources [34,35].

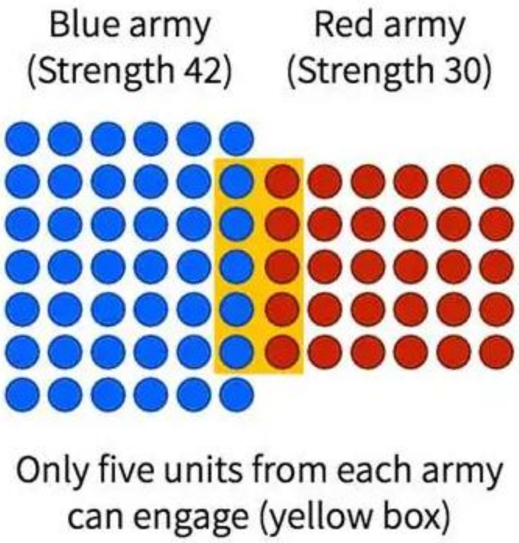

Figure 1. Sample Problem

Figure 1 assumes that each red and blue soldier in war has equal strength. Both the blue and red units cause the same number of casualties per unit of time (five – yellow area). In this case, the war will continue until the red unit is destroyed. The results are shown in Figure 2.

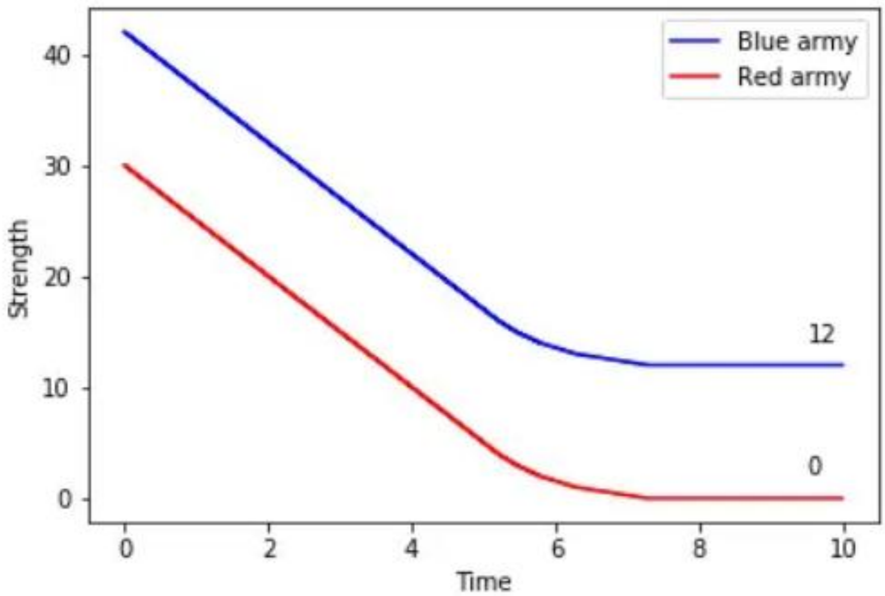

Figure 2. Linear Law Results

Figure 2 shows that the blue unit has 12 units remaining, continuing its effectiveness in the war. However, if the red unit is technically superior to the blue unit due to reasons such as better combat skills or superior equipment, the results will differ. Let us assume that in the time it takes for one blue soldier to eliminate an enemy, one red soldier can eliminate two enemies. In this case, the red side will win the war with eight survivors. The new graph is presented in Figure 3.

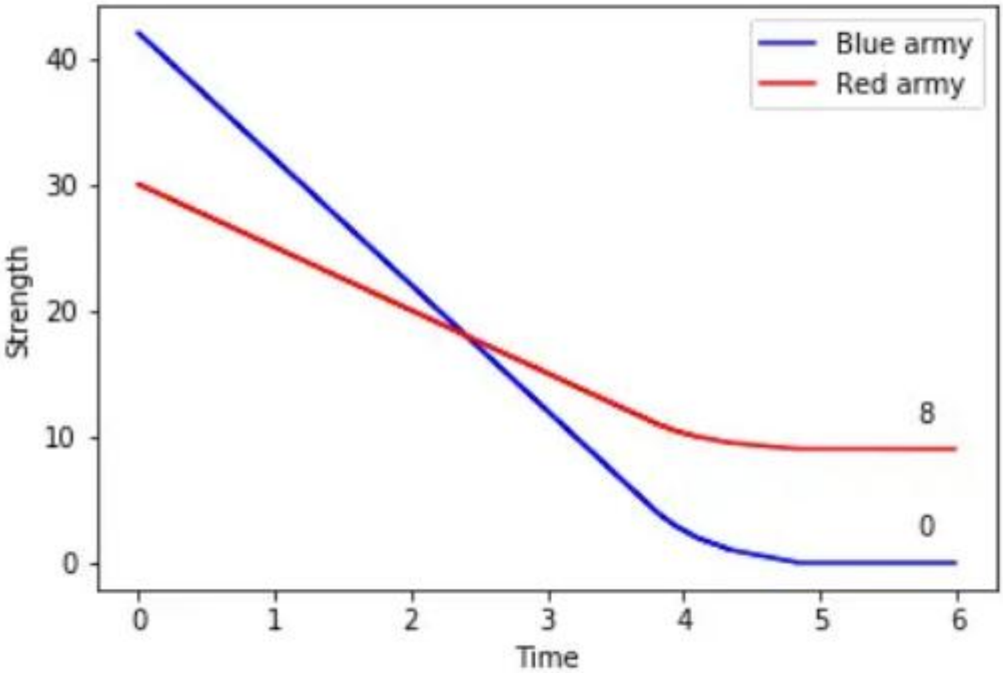

Figure 3. Change of War Powers

According to Lanchester's Linear Law, the rate of change in strength is equal to the loss rate when the war reaches equilibrium. Based on the following equation, the war will continue until the weaker side is completely defeated, unless they surrender. Even when the war ends before the complete depletion of all military forces, the remaining force of one side can be used to calculate the remaining force of the other side and, consequently, the potential losses, as shown in Equation 1.

$$\alpha * (X - X_0) = \beta * (Y - Y_0) \quad (1)$$

X_0: Initial number of red soldiers/weapons
Y_0: Initial number of blue soldiers/weapons
X: Number of red soldiers/weapons remaining at the end of the war
Y: Number of blue soldiers/weapons remaining at the end of the war
α: Destructive power of X on Y
β: Destructive power of Y on X

3.2. Square Law

In modern warfare where long-range weapons are used, the situation changes. Now, any unit can attack any target within range, and they can also be affected by the fire of multiple enemies. The side with an advantage in terms of either manpower or firepower will be able to inflict more casualties on the opposing side. In this case, Lanchester's second law, known as the Modern Warfare Law, applies, and can be calculated using Equation 2.

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$$\alpha * (X^2 - X_0^2) = \beta * (Y^2 - Y_0^2) \quad (2)$$

X_0: Initial number of red soldiers/weapons
Y_0: Initial number of blue soldiers/weapons
X: Number of red soldiers/weapons remaining at the end of the war
Y: Number of blue soldiers/weapons remaining at the end of the war
α: Destructive power of X on Y
β: Destructive power of Y on X

According to Lanchester's second law, the side with superior military equipment will suffer less damage as it wears down the enemy, and the war dynamics are proportional to the square of the sides' fighting strengths. This situation is exemplified in Figure 4, where a comparison of a blue force of 42 soldiers and a red force of 30 soldiers shows different results. While the linear model results in a continuation with 12 soldiers, the square model results in 29 soldiers.

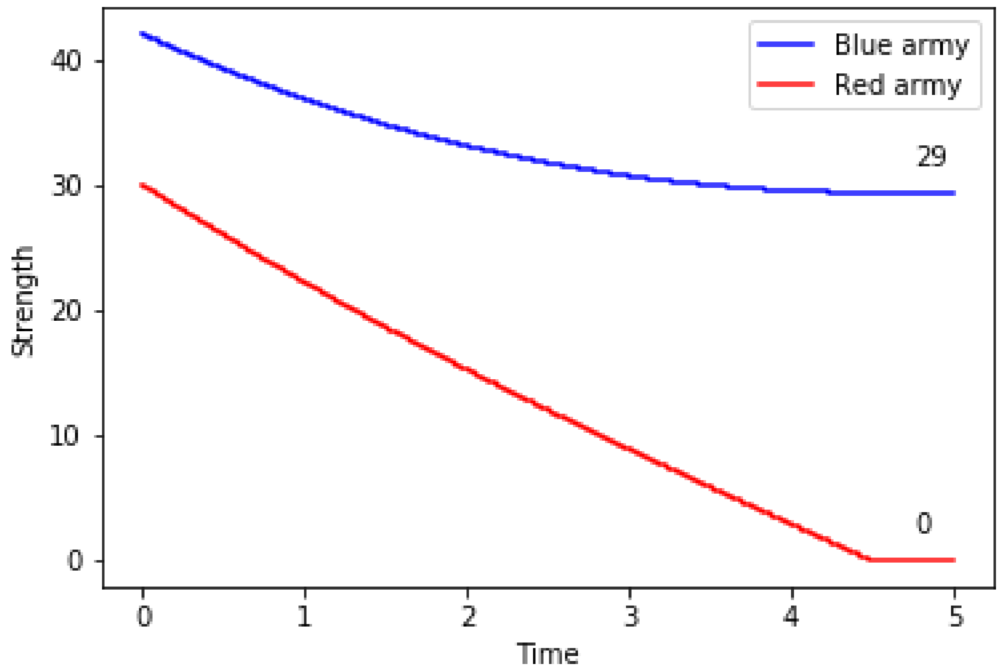

Figure 4. Square Law Results

4. Fuzzy Analytic Hierarchy Process

Fuzzy AHP is a method that combines fuzzy logic and the Analytic Hierarchy Process (AHP). Fuzzy logic is an important tool used to eliminate uncertainty and process imprecise information. The data derived from decision-makers' reasoning contains a significant amount of uncertainty. To reduce this uncertainty and provide a formal, effective result, the fuzzy method is applied. FAHP is often preferred in decision-making problems to allow for fuzzy preferences rather than relying on rigid choices. AHP, based on the decision-makers' clear opinions, may overlook important intermediary judgments. This limitation of AHP is addressed by Fuzzy AHP.

Steps in the Fuzzy AHP method [36]:

Step 1: In the first step, degree analysis is carried out. Its notation is represented as 'g_i'. The value of the degree analysis is denoted by 'M'.

M_{gi}^1, $M_{gi}^2, M_{gi}^3, \ldots, M_{gi}^n$, where i=1, 2, 3…n and j=1, 2, 3, m is represented by triangular fuzzy numbers.

Step 2: In this step, the fuzzy synthetic degree value is calculated. This value is represented by Equation 3.

$$S_i = \sum_{j=1}^{m} M_{gi}^j \otimes \left|\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j\right|^{-1} \quad (3)$$

Then, l_i, m_i, and u_i are considered as triangular fuzzy numbers, and Equations 4 and 5 are calculated.

$$\sum_{j=1}^{m} M_{gi}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j\right) \quad (4)$$

$$\left|\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j\right|^{-1} = \left(\frac{1}{\sum_{j=1}^{m} u_j}, \frac{1}{\sum_{j=1}^{m} m_j} \frac{1}{\sum_{j=1}^{m} l_j}\right) \quad (5)$$

Step 3: In this step, the probability values are defined by Equation 6. The probability values are shown by V.

$$V(M_2 \geq M_1) = \begin{cases} 1 & , M_i \geq M_j \\ 0 & , L_j \geq U_i \\ \frac{L_j - U_i}{(M_i - U_i) - (M_j - L_j)} & \end{cases} \quad (6)$$

Step 4: In the final step, the weight vector is obtained through Equations 7 and 8. The weight vector is shown by W'.

$$d'(A_1) = minV\ (S_i \geq S_k), k = 1, 2\ldots.. ,\text{n} ;\ \text{k}\neq\text{i} \quad (7)$$

$$W' = ((d'(A_1), d'(A_2), d'(A_3), \ldots.., d'(A_n))^T \quad (8)$$

5. Application and Results

In this section, an example application was conducted on the effectiveness of Anti-Tank Guided Missiles (ATGM) against a Main Battle Tank (MBT), and the methodology is shared in Figure 5.

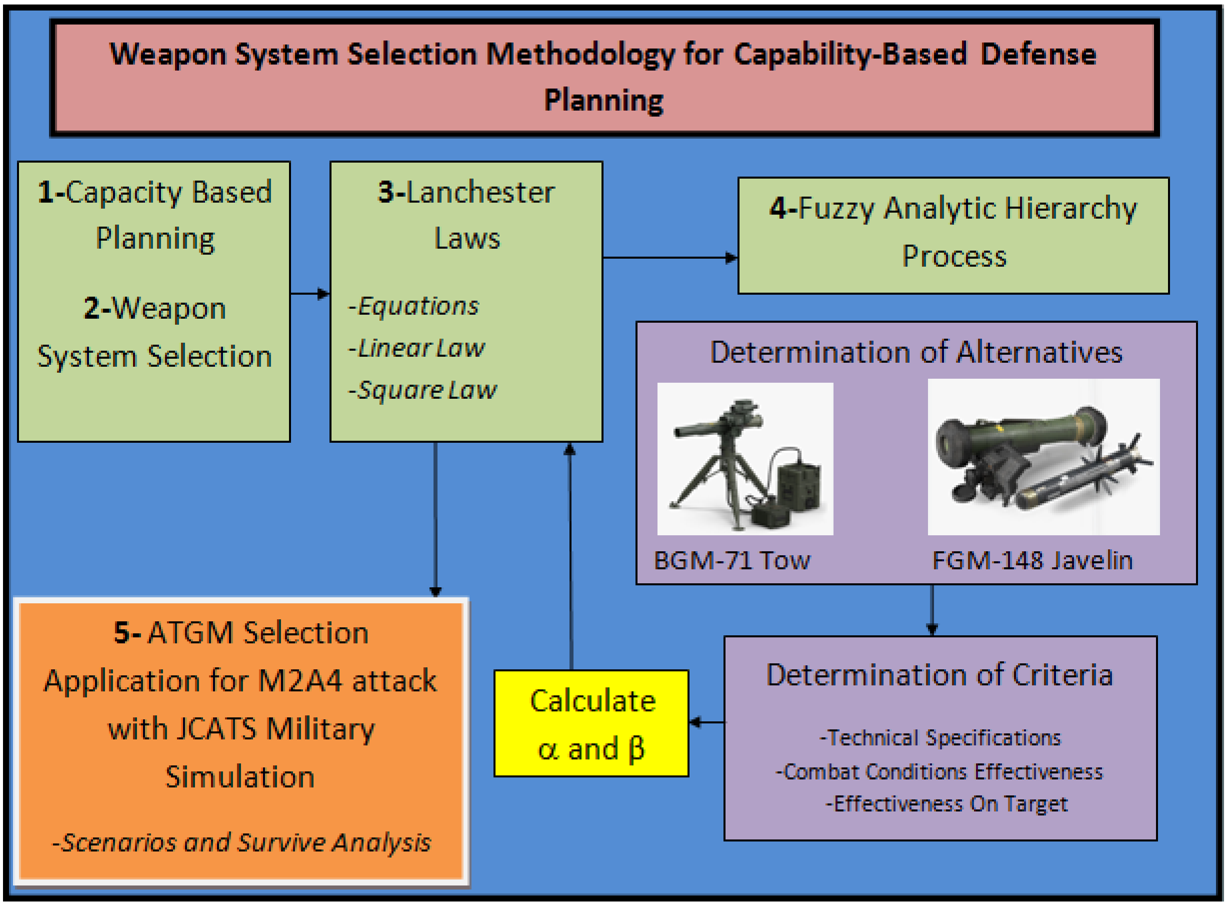

Figure 5. Methodology

For the application, the JCATS military simulation program was used. A screenshot of the blue and red units from the example scenario is provided in Figure 6 [37-39].

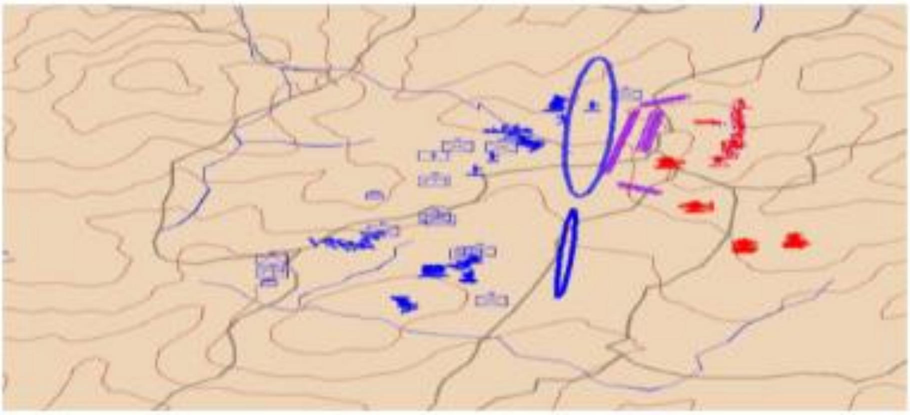

Figure 6. A Partial JCATS Interface for A Simulation Scenario

Since the JCATS program was utilized in the study, the M2A4 tank was selected as the MBT alternative. The purpose of the scenario is to make decisions in capability-based defense planning to choose a missile system capable of targeting tanks from the top. For this reason, the ATGM systems of the X and Y forces, namely the BGM-71 Tow and FGM-148 Javelin, which shared the first two positions in the study by Erdal et al. (2023), are treated as generic alternatives [5]. The α ve β weights for the ATGM systems are calculated by considering technical specifications, combat conditions effectiveness, and effectiveness on target criteria, using FAHP [5, 40]. The values in Table 1 are shared as an example of the calculations in Section 4.

Table 1. Fuzzy Weight Values and Normalized Weight Values

ATGM-1	L	M	U	M_i	N_i
Technical Specifications	0.132	0.318	0.788	0.413	0.351
Combat Conditions Effectiveness	0.226	0.461	0.788	0.492	0.418
Effectiveness On Target	0.132	0.221	0.461	0.271	0.231

Since the final weights are calculated based on 3 criteria, the values are divided by the number of criteria and summed for each weapon alternative. For example, for Table 1, Ni/3 values are calculated as 0.117, 0.139 and 0.077, and summed as 0.333. This reflects the α value. The β value is similarly calculated and found to be 0.299. The results of the analysis with 30 and 42 ATGMs in 13 different scenarios are presented in Table 2.

Table 2. Scenario Results

	B	R	Blue Analysis	Red Analysis	Winner
S1	30	42	9.99	12.558	Red
S2	31	42	10.323	12.558	Red
S3	32	42	10.656	12.558	Red
S4	33	42	10.989	12.558	Red
S5	34	42	11.322	12.558	Red
S6	35	42	11.655	12.558	Red
S7	36	42	11.988	12.558	Red
S8	37	42	12.321	12.558	Red
S9	38	42	12.654	12.558	Blue
S10	39	42	12.987	12.558	Blue
S11	40	42	13.32	12.558	Blue
S12	41	42	13.653	12.558	Blue
S13	42	42	13.986	12.558	Blue

According to these results, the blue force wins in 5 of the 13 scenarios. The superiority of ATGM shifts in the nineth scenario. The results obtained with this generic data show the superiority of the BGM-71 Tow alternative. The results will change when criteria and expert evaluations are altered. The proposed methodology should be tested with real data and experts. The obtained results are consistent with the study by Erdal et al. (2023) [5]. It has been demonstrated that the FMCDM procedure can be supported by Lanchester equations.

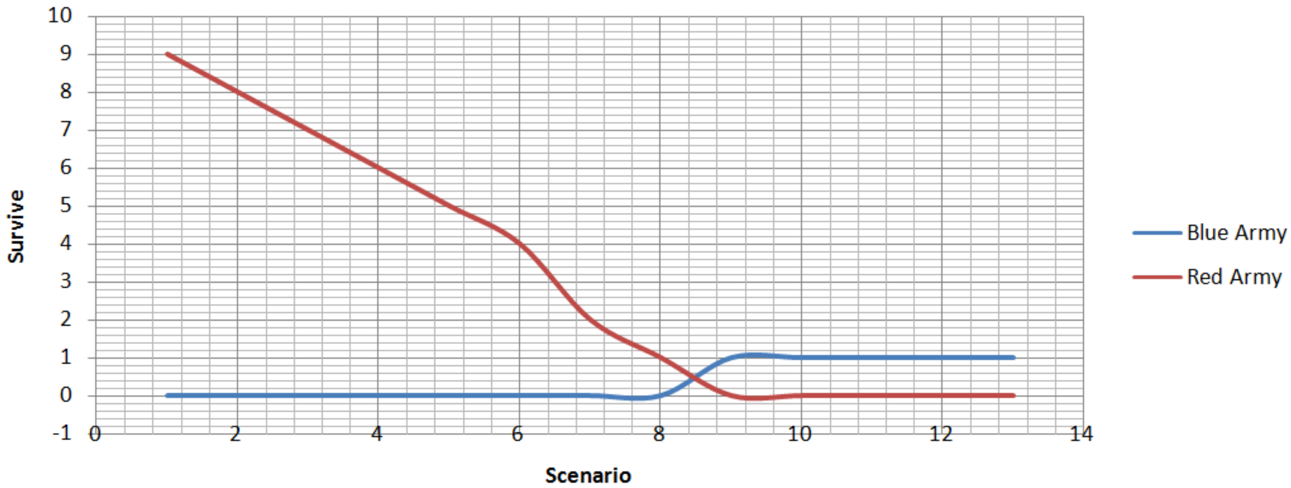

Figure 7. Survive Analysis with Square Law in Scenarios

6. Conclusions

In conclusion, the selection of weapon systems within the framework of capability-based defense planning is one of the most critical elements of being prepared for future combat environments. Identifying flexible and versatile systems capable of adapting to changing threats directly impacts the success of military operations. In this process, simulations and the testing of different scenarios provide decision-makers with a scientific and data-driven approach, ensuring the most efficient use of resources. Furthermore, considering factors such as interoperability with allies, sustainable logistics, and cost-effectiveness, the long-term strategic contributions of weapon systems, not just their technical capabilities, gain importance. Therefore, making correct weapon system selections within the capability-based planning framework enhances deterrence and ensures future operational superiority.

At this point, not only technical and cost analyses but also analytical decision-making methods play a significant role in the selection of weapon systems. Various analytical approaches, especially FMCDM, provide decision-makers with a broader perspective in defense planning processes where uncertainties are high. Evaluating different weapon systems in terms of operational effectiveness, logistical sustainability, and strategic alignment goes beyond one-dimensional decision-making processes. This allows military capabilities to gain flexibility against long-term threats and ensures optimal resource allocation. The objective evaluation framework provided by analytical methods contributes to making defense planning more predictable and effective, supporting the success of the capability-based approach.

In this context, among the evaluations conducted within the analytical framework, the Lanchester equations remain significant models that continue to retain their validity. The Lanchester equations mathematically model the dynamics of combat between two opposing forces, enabling the analysis of the impact of variables such as force size and firepower on the outcome of the battle. These equations, widely used in traditional warfare scenarios, also provide valuable insights in modern battlefields involving precision-guided weapon systems. Particularly in asymmetric warfare and multi-threat environments, the Lanchester models, when considered alongside other analytical decision-making methods, continue to serve as a powerful forecasting tool for determining the effective use of forces and optimizing combat plans.

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Conflicts of Interest
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