Knowledge and Decision Systems with Applications

www.kadsa.org
eISSN: 3079-6318

A Decade of Picture Fuzzy Sets in Multi-Criteria Decision-Making: A Comprehensive Review of Trends, Gaps, and Future Directions

Chukwudi Obinna Nwokoro[1,*] and Paul Augustine Ejegwa[2]

[1] Department of Computer Science, Nigerian British University, Asa, Abia State, Nigeria
[2] Department of Mathematics, Joseph Sarwuan Tarka University, Makurdi, Nigeria

ARTICLE INFO

Article history:
Received 24 March 2025
Received in revised form 5 May 2025
Accepted 9 May 2025
Available online 10 May 2025

Keywords:
Picture Fuzzy Sets; Multi-Criteria Decision-Making; Uncertainty Management; Aggregation Operators and Artificial Intelligence

ABSTRACT

Picture fuzzy sets (PFSs) are extensively utilized in medical diagnostics and multi-criteria decision-making (MCDM) due to their enhanced flexibility, distinguishing capability, and applicability in handling uncertainty and hesitation, particularly in complex domains such as healthcare, transportation, environmental decision-making, artificial intelligence (AI), and machine learning. Guided by four research questions, this review employed descriptive statistics to analyse the extent of research focused on PFSs. An intensive literature search was performed across leading publishers, including IEEE Xplore, SpringerLink, ScienceDirect, the Association for Computing Machinery (ACM), and the Multidisciplinary Digital Publishing Institute (MDPI). Findings revealed significant scholarly efforts to adopt PFSs in healthcare (22.6%), transportation (24.5%), environmental decision-making (7.5%), AI and machine learning (5.7%), and other domains (39.6%). Research from 2013 to 2024 demonstrated notable advancements in mathematical operations and extensions (25%), aggregation operators and similarity measures (30%), hybrid approaches and MCDM applications (20%), domain-specific implementations (15%), and theoretical developments (10%). MCDM emerged as a prominent tool for enhancing decision-making across diverse fields. The study highlights the need to explore additional areas of application for PFSs, particularly in refining distance measures to further enhance their utility.

1. Introduction

Approximation methods (AM) remain exceptionally functional in handling uncertainty, vagueness, and imprecision in data or decision-making processes. Its application in real-world problems cannot be overemphasized because it helps to give more insights where precise information is unavailable or incomplete [1-6]. Fuzzy set (FS) is a type of AM, which has undergone a lot of paradigm shifts starting from the traditional sets which help to classify elements on where they belong to or not, which was becoming too rigorous for the tangible concern.

* *Email:* concok02@gmail.com
https://doi.org10.59543/kadsa.v1i.14051

In contrast, FS permit degree of membership to enable the modeling of concepts that are not true or false but approximate (for instance, predicting the weather, which can be "hot," "warm," or "cold" the fundamental question is to what degrees is it) rather than having a single exact value [7,8]. This gives rise to approximate reasoning that tries to mimic human decision-making, among others, precise and incomplete data interpretation, most importantly where
most judgments are not binary but are based on partial truths such as a person can be described as someone tall, which can be represented with a fuzzy degree of truth rather than a precise height cut-off. In some systems, adopting particular mathematical models may be difficult to implement due to complexity or lack of accurate data, so employing a FS becomes paramount to creating an approximation model, which uses membership functions to represent uncertainty and approximate the behaviour of the complex systems.
Estimated inference remains a product of fuzzy sets, is derived from Fuzzy logic and is utilized to alleviate the burden of controlling systems, such as thermostats [9], washing machines [10], [11], and AI systems [12,13], in approximating human reasoning. Zadeh [14] introduced FS by defining a membership function $\mu_H(x)$ that allocates each element x with degree of membership in the set H, where $\mu_H(x) \in [0, 1]$. This can be used to represent incomplete membership, hereby making FSs useful in handling uncertainty and vagueness in data, which was widely applied in control systems, such as automated vehicle control system like Anti-lock Braking System (ABS), where strict control is not necessary, and smooth evolutions are more effective. Despite the implementation of FS, it still was not able to model non-membership, limiting their ability to handle more complex uncertainty.

In 1986, Atanassov [15] introduce intuitionistic fuzzy sets (IFS) to address both membership $\mu_H(x)$ and non-membership $v_H(x)$, implies that $\mu_H(x) + v_H(x) \leq 1$, with the residual $1 - \mu_H(x) - v_H(x)$ representing the degree of hesitation. This framework provides an improved way to represent uncertainty, making IFS highly applicable in medical diagnosis systems [16-18], where both profit potential and risk of loss indication, as well as ambiguity, needs to be accounted for; due to increase in computational complexity and continuous representational power. Cuong [19] proposed Picture Fuzzy Sets (PFSs), as a way of extending IFS to consider neutrality function $\pi_H(x)$ where $\mu_H(x) + v_H(x) + \pi_H(x) \leq 1$. PFS can model situations with greater ambiguity by considering neutral states, offering more expressive power. Existential application of PFSs include decision-making in various fields of human endeavour, among others are social sciences and sentiment analysis, where neutral opinions or indecision must be accounted for alongside positive and negative sentiments. However, this added expressiveness, increases the complexity of decision-marking and computational costs of operations. Furthermore allow more granular and realistic assessments of environment health, where conditions may not always be clearly good or bad but may also exist in neutral or indeterminate state. It is against this backdrop that this work will be guided by the following research questions (RQ): RQ1 to investigates the existing reviews of PFSs since their introduction; RQ2 examines the domains where PFSs have been effectively applied; RQ3 identifies the most commonly used parameters for tuning PFSs; and finally, RQ4 examines the extent to which new methodologies have been proposed to enhance the theory of PFSs.
The primary objective of this study is to conduct a comprehensive analysis of PFSs by reviewing existing literature, examining their applications across various domains, exploring common tuning parameters, and evaluating new methodologies proposed to enhance their theoretical framework. Subsequent sections of this work will be structured as follows: Section II will cover the methodology, Section III will present results from the RQs with discussions and, Section IV will focus on the conclusion and recommendations.

2. Methodology

Critical review (CR) acts as a roadmap for scholarly inquiry, allowing researchers to expand upon existing knowledge, tackle unresolved issues, and offer deeper insights into the domain systematically and informally. This approach enables us to synthesize existing studies and identify key trends, strengths, and limitations in the field since the introduction of PFSs. We conducted a comprehensive search across several databases, including "IEEE Xplore", "SpringerLink", "ScienceDirect,", "Association for Computing Machinery (ACM)", "Multidisciplinary Digital Publishing Institute (MDPI)", and several journals using targeted keywords such as "Picture Fuzzy Sets," "applications," "distance and similarity measures" and "methodologies", "Multi-Attribute Decision-Making (MADM)".

To ensure a thorough and unbiased analysis, we establish inclusion criteria which focus primarily on works related to PFSs. Specifically; we include papers that discussed theoretical advancements, practical applications, or innovative methodologies in PFSs, with a time frame of 2013 to 2024 to capture recent developments. Only research published in peer-reviewed journals was considered to ensure academic rigour and quality. Additionally, we restricted our review to studies published in English to avoid translation issues and ensure accessibility. We also included empirical, theoretical, and methodological studies that contribute to understanding PFSs, their applications, or their enhancements in decision-making, uncertainty management, or related domains. Furthermore, only full-text studies were selected for a thorough analysis and critique.

Our review process was further refined by a set of exclusion criteria that ensured the thoroughness and quality of our review. These criteria eliminated studies that did not directly address PFSs, articles from non-academic or non-peer-reviewed sources (e.g., blogs, opinion pieces, popular magazines), studies published in languages other than English, and duplicated or redundant research. Papers that were not adequately reported or older studies significantly outdate by more recent advancements were also excluded to maintain the quality of the review, to ensure the quality of the review, studies that were poorly reported or rendered obsolete by more recent advancements were excluded. The selection process was mapped using a flowchart to depict the logical steps followed in this study.

Each selected work was critically assessed for its contributions to understanding PFSs, particularly emphasizing the effectiveness of PFS applications in practical scenarios. This critical review methodology aims to highlight the current state of PFS research and identify areas for future exploration and development.

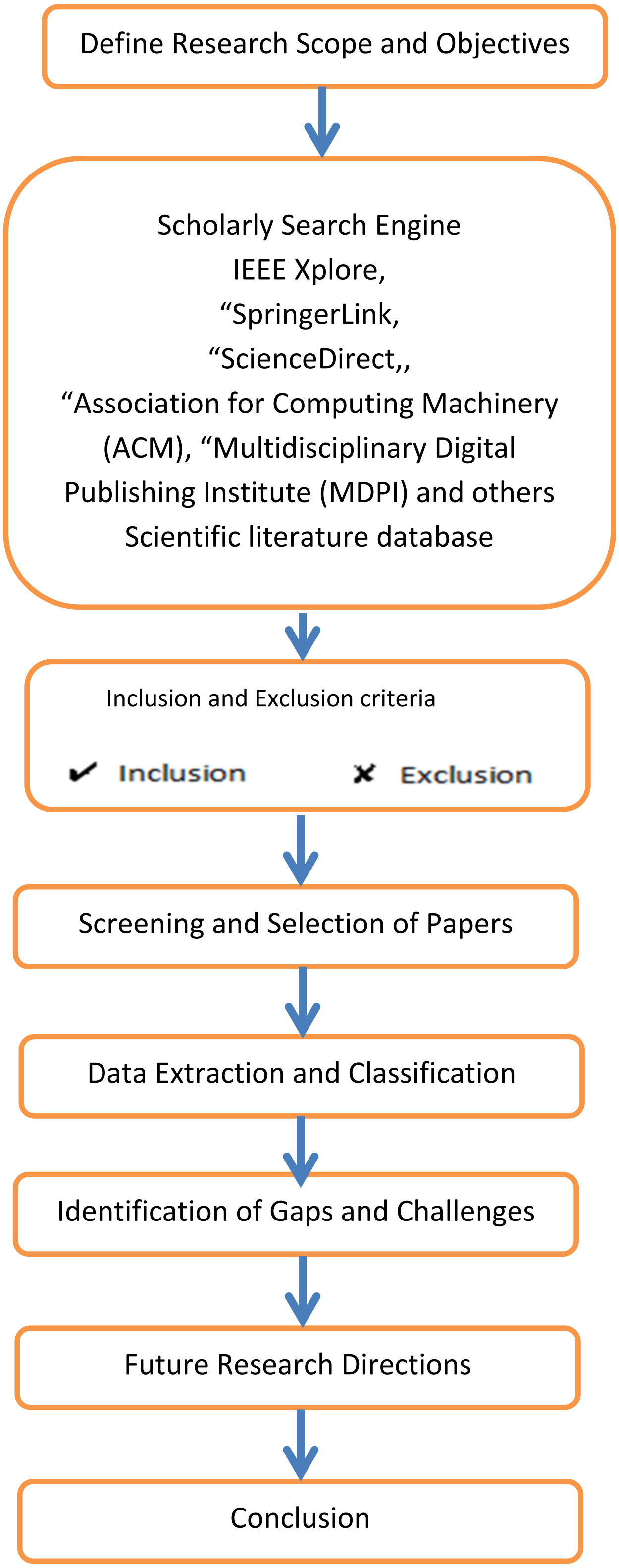

Define Research Scope and Objectives
Scholarly Search Engine
IEEE Xplore,
“SpringerLink,
“ScienceDirect,,
“Association for Computing Machinery
(ACM), “Multidisciplinary Digital
Publishing Institute (MDPI) and others
Scientific literature database
Inclusion and Exclusion criteria
Inclusion
Exclusion
Screening and Selection of Papers
Data Extraction and Classification
Identification of Gaps and Challenges
Future Research Directions
Conclusion

Fig 1: Study Selection Procedure on Picture Fuzzy Sets Review

Figure 1 is a flowchart illustrating a review approach encompassing the identification of relevant academic databases, rigorous screening based on inclusion criteria, thematic categorization of studies, and the synthesis of findings.

3. Results

3.1 RQ1: Explores the Existing Reviews of PFSs since Inception

We start with RQ1, which borders about exploring the existing reviews of PFSs since their introduction. Several studies have been conducted to highlight the relevance and significance of Picture Fuzzy Sets (PFSs), these studies not only highlight the theoretical advancements but also demonstrate the practical applications of PFSs in decision-making and uncertainty handling. A comprehensive summary of these contributions is presented in Table 1, providing a detailed overview of key findings and developments in the field.

Table 1: Comprehensive Overview of Contributions across Key Domains and Methodologies Employed

Year	Author(s)	Contribution	Area	Key Approach
2013	Cuong *et al*[19]	Introduced PFSs, extending traditional FSs to handle uncertainty, indeterminacy, and neutrality.	General theory	Basic mathematical structure of PFSs and initial operations
2015	Zhao *et al*[20]	Developed dynamic PFSs for distance measures and parametric similarity measures to enhance multi-attribute decision-making (MADM).	Decision-making (MADM)	Distance and parametric similarity measures
2016	Duong and Thao[21]	Proposed a dissimilarity measure for PFSs and applied it to MCDM.	MCDM	Dissimilarity measures
2017	Van Dinh *et al*[22]	Implemented distance and dissimilarity measures for PFSs to improve decision-making processes.	Decision-making	Combination of distance and dissimilarity measures
2018	Palash Dutta[23]	Applied PFSs in medical diagnosis, particularly focusing on tropical diseases.	Medical diagnosis	Application of PFSs to real-world health problems
2019	Athira[24]	Introduced entropy and distance measures for decision support systems, enhancing the handling of uncertainty.	Decision Support Systems	Entropy and distance measures
2020	Riaz *et al*[25]	Introduced bipolar PFS operators with new distance measures for MCDM.	MCDM	Bipolar PFS operators
2020	Singh and Ganie[26]	Utilized PFS similarity measures for pattern recognition and clustering.	Pattern recognition and clustering	Similarity measures

2021	Ahmed and Dai[27]	Proposed similarity measure and multi-expert TOPSIS approach for PFS and picture m-polar fuzzy sets.	Decision-making (TOPSIS)	Similarity measures and multi-expert approaches
2021	Mahmood *et al*[28]	Compared cosine function with TOPSIS method for strategic decision-making using picture hesitant fuzzy sets.	Strategic decision-making	Cosine function and hesitant fuzzy sets
2021	Cao[29]	Developed a new TOPSIS method using PFSs for identifying the ranking order of determinants in mobile multimedia healthcare service adoption.	Healthcare	TOPSIS and ranking techniques
2021	Khalil *et al*[30]	Developed fundamental operations for interval-valued PFSs and applied them to decision-making problems.	Decision-making	Interval-valued PFSs
2022	Devi *et al* [31]	Applied dimensionality reduction methods in PFSs to improve environmental decision-making.	Environmental decision-making	Dimensionality reduction
2022	Umar and Saraswat [32]	Proposed a novel divergence measure for PFSs	Decision making	Machine learning
2023	Hussain *et al* [33]	A reliable decision-making framework for supplier selection utilizing	MADM	Analysing the distance measures and algorithms
2024	Dhumras *et al* [34]	Applied similarity measures of complex PFSs (CPFSs) to pattern recognition and medical diagnosis	Similarity Measures	Proposed a novel approach for CPFSs
2024	Khan *et al* [35]	Developed a decision-making framework for pattern recognition in healthcare management.	CPFSs	Proposed an enhanced approach for healthcare systems to deliver personalized medicine.

From Table 1, it is imperative to note that works carried out in PFSs from 2013 to 2024 have been applied across several domains, from improving MADM to evolving healthcare applications. Early work laid the groundwork for handling uncertainty, indeterminacy, and neutrality in PFSs. Subsequent improvements introduced dissimilarity and dynamic measures for decision-making procedure, entropy-based improvements for resolution support systems, and applications in medical diagnosis. More recent innovations include novel approaches to strategic decision-analysis, multifaceted PFSs in structure identification, and frameworks for adapted medication in patient care, showcasing the various and growing relevance of PFSs. However, its impact cannot be overstated, as it has been applied in numerous fields, including healthcare, pattern recognition, transportation, environmental decision-making, and artificial intelligence. Given its significant contributions, exploring the extent of research conducted across these areas is essential in Table 2 to help showcase the effort made by scholars.

3.2 RQ2: Examines the Domains of Application of PFSs

We examine the domains where PFSs have been effectively applied and the outcomes are presented in Table 2.

Table 2: Analysis of Insights from Works Conducted across Various Domains

S/No	Application area	Cited works by Authors	Summary
1.	Healthcare	Bani-Doumi *et al* [36], Cao and Shen[37], Van Pham *et al* [38], Dutta[39], Haktanır *et al* [40], Senapati and Chen[41], Pham *et al* [42], Asghar *et al* [43], Haktanır and Kahraman [44], Tchier *et al*[45], Almulhim and Barahona[46], Monika *et al* [47]	In this section, a wide range of studies have been directed toward personalized medicine to improve medical decision-making, treatment protocols, and patient care. The focus is on managing uncertainty to enhance treatment plans and enable more accurate diagnoses.
2.	Transportation	Bakioglu [48], Mehmood and Bashir [49], Pham *et al* [50], Kutlu-Gündoğdu *et al* [51], Zuo *et al* [52], Simić *et al* [53], Duleba *et al* [54], Jovcic *et al* [55], Kou *et al* [56], Švadlenka *et al* [57], Riaz and Farid [58], Xiao *et al*[59] , Akram *et al* [60]	In this domain, most studies emphasize that PFSs effectively address transportation challenges, including resource allocation, congestion management, and sustainability.
3.	Environmental Decision-Making,	Garg *et al* [61], Chitra and Maheswari [62], Simić *et al* [53], Biswas *et al* [63]	Several studies have primarily focused on climate change mitigation, resource management, and pollution control, utilizing PFSs to demonstrate their effectiveness in managing complex environmental systems where MCDM and uncertainty play a significant role.
4.	Artificial Intelligence & Machine Learning	Memiş [64], Tamilselvan [65], Xu *et al* [66],	It explores the integration of artificial intelligence and machine learning to enhance decision-making among both healthcare and non-healthcare professionals, highlighting the power of data analysis and predictive modelling in uncovering patterns and trends within computational systems.

5.	Others	Zhu *et al* [67], Razzaq and Riaz [68], Imran-Harl *et al* [69], Pham *et al* [50], Zhu *et al* [70], Yang *et al* [71], Singh and Kumar [72], Dutta *et al* [73], Ateş and Akay [74], Khoshaim *et al* [75], Kamacı *et al* [76], Kumar and Bisht [77], Kumar and Bisht [77], Gül *et al* [78], Yu *et al* [79], Arya and Kumar [80], Aydoğmuş *et al* [81], He and Wang [82], Kahraman [83], Simić *et al* [84], Sindhu *et al* [85], Göçer [86], Zhao *et al* [87], Son *et al* [88].	This section highlights that extensive research has been applied to finance, education, manufacturing, and power supply areas. It further demonstrates the versatility of PFSs in addressing real-world challenges, including planning, risk assessment, and policy implementation.

Based on our findings, the analysis reveals that a significant portion of research has been conducted across diverse areas of human endeavor. As shown in Table 2 and Figure 2, 22.6% of the works are focused on healthcare, 24.5% on transportation, 7.5% on environmental studies, 5.7% on AI and machine learning applications and 39.6% are distributed across other fields.

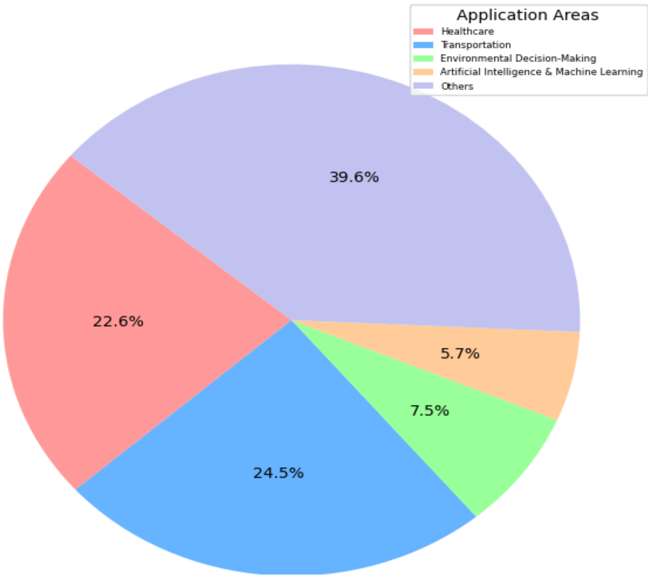


Fig. 2: Distribution of research focused on picture fuzzy sets

These results connote that PFSs remain underutilized but are evolving as an encouraging tool in soft computing. Furthermore, several authors, including Ateş and Akay [74], Kumar and Bisht, [77], Qiyas *et al* [89] and Simić *et al* [84] have explored PFSs in various contexts. For instance, Arya and Kumar [80] applied MCDM techniques such as TOPSIS and other numerical methods to enhance decision-making by addressing uncertainty, indeterminacy, and neutrality [90]. Other researchers, like Singh and Ganie [26], Verma and Rohtagi [91], Thong and Son [92], and Fan *et al*[93], have focused on hybrid approaches for pattern recognition. This highlights the need to assess the methodologies adopted in employing PFSs and their varying levels of sophistication.

3.3 RQ3: Investigating the New Methodologies for the Enhancement of the theory of PFSs

In recent years, researchers have introduced new refinements, aggregation techniques, hybridization methods, and conceptual advancements in PFSs. In this section, we will explore these aspects in detail.

a) Mathematical Operations and Extensions

Several scholars have proposed new operations to expand the basic structure of PFSs. For instance, Zuo *et al* [52] introduced isomorphic picture fuzzy graphs that include composition, Cartesian products, and lexicographic products, which have been applied in social networks and later enhance medical diagnosis during COVID-19 [94]. Cuong and Kreinovich [95] explored operations on PFSs, although their work has yet to be implemented in specific domains. Additionally, Khalil *et al* [30] proposed some operations on interval-valued PFSs within soft sets, applied in customer care centres, and Akram *et al* [60] presented a method for solving linear programming problems using PFSs in an auto workshop setting. These advancements illustrate how new mathematical structures within PFSs are improving their adaptability and precision.

b) Aggregation Operators and Similarity Measures

A wide range of aggregation operators has been introduced to better handle uncertainty and indeterminacy. Hussain *et al* [96] proposed aggregation operators like the picture fuzzy Schweizer-Sklar prioritized average and geometric operators for information selection. Similarly, Arya and Kumar [80] introduced a probabilistic framework for picture fuzzy entropy to enhance election results. Özer [97] compared Continuous Interval-Valued Fuzzy Hybrid Power Averaging (CIFHPA) and Continuous Interval-Valued Fuzzy Hybrid Power Ordered Averaging (CIFHPOA) operators to improve decision-making in complex PFS scenarios. These aggregation techniques contribute to enhanced decision-making by offering more refined methods for dealing with fuzzy, uncertain data.

Moreover, Verma and Rohtagi [91] developed a similarity measures for PFSs in pattern recognition and medical diagnosis, while Ganie and Singh [98] proposed a picture fuzzy distance measure to support decision-making in commercial banking. These innovations have significantly improved decision-makers' handling of indeterminacy and neutrality in various fields, from medical diagnostics to finance.

c) Hybrid Approaches and MADM

Researchers have increasingly adopted hybrid approaches by integrating PFSs with MCDM methods. Arya and Kumar [99] suggested a TODIM-VIKOR approach based on entropy and Tsalli divergence measures for PFSs, which improved decision-making accuracy in high-stakes environments. Similarly, Ma *et al* [100] used interval-valued PFSs with a MCDM approach for concept evaluation, while Sindhu *et al* [101] adopted bipolar PFSs for investment plan selection. These hybrid models provide more comprehensive solutions for decision-making problems involving multiple, conflicting criteria. One significant operator used in the hybrid

MADM model is the t-Norm and t-Conorm, particularly Frank t-Norm and t-Conorm, which help improve decision-making in handling uncertainty and provide flexible and parameter-dependent methods. These operators sum localized fuzzy aggregation techniques by providing tunable control over the union (t-Conorm) and intersection (t-Norm) of picture fuzzy membership degrees, thereby giving a good construct for decision-making.

Some work, such as Boixader and Recasens[102], Seikh and Manda[103]l, Mehmood and Liu[104], used these operators to enable a more adaptable and accurate representation of expert opinions in complex evaluations. Also, these operators stand out in scenarios where standard arithmetic mean or weighted averages fail to capture the bound between multiple criteria. Additionally, Sarfraz [105] showcase the strength of t-Norm-based aggregation in handling robust contradictory or hesitant opinions while the t-Conorm was used to fuse the consensus building in multiple group decision-making contexts.

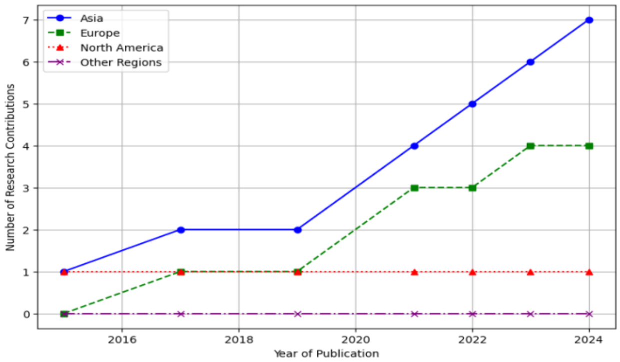
d) Domain-Specific Applications

New methodologies have been applied across various domains, indicating the growing practicality of PFSs. Ahmad *et al* [106] utilized picture fuzzy similarity measures for pattern recognition in building materials, while Dhivya *et al* [107] employed PFS-based differential equations to enhance drug distribution in the human body. Song and Ding [108] applied the Choquet integral to develop picture fuzzy correlated averaging (PFCA) operators in the high-tech industry. Mahmood *et al* [109] used complex picture fuzzy N-soft sets to predict outcomes in areas like e-waste recycling and FIFA World Cup predictions. These applications demonstrate how PFSs are being used to solve real-world problems, particularly in medical diagnosis, investment, and environmental management.

e) New Theoretical Contributions

In addition to practical applications, theoretical contributions have been made to expand the conceptual framework of PFSs. Zhang [110] redefined basic operations for PFSs to extend their theoretical principles, although these advancements still need to be applied in any domain, suggesting room for further research. Similarly, Bo and Zhang [111] proposed a picture relations to assess capital investment risk factors, while Cuong and Thong [112] introduced a concepts such as Picture Fuzzy Rough Soft Sets and Picture Fuzzy Dynamic Systems, providing novel frameworks for future exploration.

In summary, most of the reviewed works highlight new methodologies to enhance the theory of PFSs in recent years. These progressions primarily focus on healthcare, finance, and social networks, with innovations in aggregation operators, mathematical refinements, hybrid approaches, and domain-specific applications. While some contributions remain theoretical, many have been successfully implemented, demonstrating that PFSs are evolving into robust tools for managing uncertainty. These innovations strengthen the original theory and underscore the need for broader practical applications across diverse fields, which were represented in visually in Figure 2 and 3.



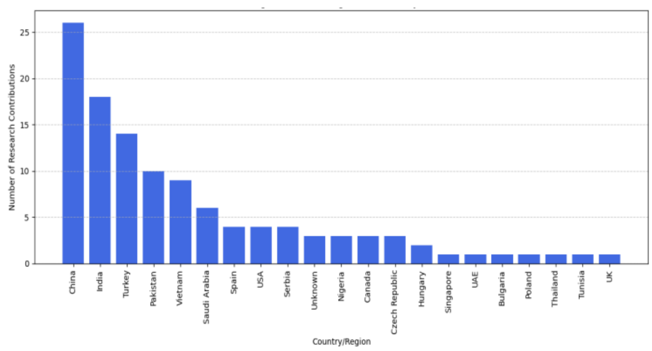


Fig. 4: Contribution to picture fuzzy sets research

Fig. 3: Industrial application in PFSs research by region

Many scholars have explored picture fuzzy sets across various countries, increasing awareness of this research area. However, Figure 3 reveals that the highest number of studies originate from China, positioning it as the leading contributor, followed by India in second place, Turkey in third, Pakistan in fourth, and Vietnam in fifth. Saudi Arabia ranks sixth, followed by Spain, the USA, and Serbia. Nigeria, Canada, and the Czech Republic share the next position, followed by Hungary. The final group includes Singapore, the UAE, Bulgaria, Poland, Thailand, Tunisia, and the UK.

This distribution highlights a significant global effort to integrate picture fuzzy sets into decision-making processes across various domains. Figure 4 illustrates the trend analysis of contributions by region, showing that Asia leads in research output, followed by Europe in second place, and North America in third. From 2013 to 2024, the research focus has been primarily on transportation, sustainability, and logistics, with a notable increase in publications in 2021, 2022, and 2023.

Although North America contributes to PFS research, its adoption for industrial applications is not expanding as rapidly as in Asia and Europe. The line representing "Other Regions" remains consistently flat at zero, indicating no significant industrial research contributions from regions outside Asia, Europe, and North America.

This could be as a result of lack of research adoption in Africa, South America, and Oceania, potentially due to limited research funding or focus in these regions, which in line with the works of Odekunle *et al*[113], Nijman-Ross *et al*[114] and Bailey *et al* [115] suggested the need for stakeholders to embrace research integration to improve decision making among experts.

3.4 RQ4: Identifying the most commonly used Parameters for Tuning PFSs

Based on the studies reviewed in RQ3, we found it prudent to analyse the frequency of parameters employed by scholars to enhance the effectiveness of PFSs. A summary of this analysis is presented in Table 3.

Table 3: Commonly used Parameters for Tuning Picture Fuzzy Sets (PFSs) and Their Applications

Parameter	Authors	Purpose	Applications
Mathematical Operations (e.g., composition, Cartesian products, lexicographic products)	Zuo *et al* [51], Cuong and Kreinovich [94], Khalil *et al* [30], Akram *et al* [59]	To enhance adaptability, precision, and solve linear programming problems using PFSs	Social networks, customer care centres, auto workshops, medical diagnosis during COVID-19
Aggregation Operators (e.g., Picture Fuzzy Schweizer-Sklar prioritized average, geometric operators)	Hussain *et al* [95], Arya and Kumar [79], Özer [96]	Toward better handle uncertainty and indeterminacy, improve decision-making accuracy	Election results, complex PFS scenarios, decision-making in various domains
Similarity and Distance Measures (e.g., distance measures, entropy measures)	Verma and Rohtagi [90], Ganie and Singh [97], Arya and Kumar [79]	Improve handling of indeterminacy and neutrality in decision-making	Medical diagnosis, pattern recognition, commercial banking, elections
Hybrid Approaches (e.g., TODIM-VIKOR, entropy-based, Tsalli divergence)	Arya and Kumar [98], Ma *et al* [99], Sindhu *et al* [100]	To integrate PFSs with MCDM methods for more comprehensive decision-making	High-stakes environments, concept evaluation, investment plan selection
Similarity Measures for Domain-Specific Applications (e.g., Choquet integral, picture fuzzy correlated averaging)	Ahmad *et al*, Song and Ding [102], Dhivya *et al*[101], Mahmood *et al* [103]	Apply PFSs in specialized fields, enhancing decision-making and solving real-world problems	Building materials, drug distribution, e-waste recycling, FIFA World Cup predictions
New Theoretical Contributions (e.g., Picture Fuzzy Rough Soft Sets, Picture Fuzzy Dynamic Systems)	Zhang [104], Bo and Zhang [105], Cuong and Thong [106]	To expand the theoretical framework and introduce novel concepts for future exploration	Capital investment risk assessment, theoretical extensions of PFSs

Table 3 illustrates how these parameters are extensively utilized to improve the precision, adaptability, and applicability of PFSs across various domains, enhancing decision-making processes among experts. Additionally, there are several key researchers and pioneers promoting the application of PFSs, based on this it are imperative to illustrate authors and their corresponding citation counts because one work leads to another.

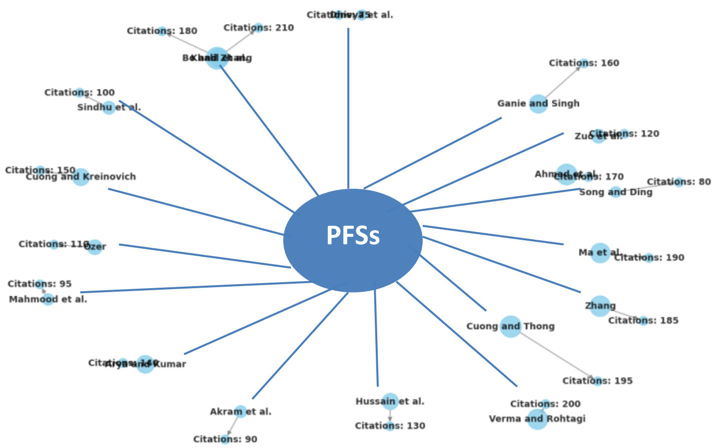


Fig. 5: Authors citation-weight relationship network diagram

Figure 5 represents a network diagram and its connections. Each author shows the node connected to the citation node and the weight assigned to the author. Cuong and Thong [106] and Bo and Zhang [105] show higher weights appear larger, suggesting their contributions are more influential or widely cited. Citation nodes are showcased as secondary references to illustrate citation distribution.
The structure primarily portrays collections of authors' strengths of wavering citations, enlightening vastly cited scholars against those with lower or moderate citations. One hallmark of this visualization is that it helps spot the key thought leaders, state-of-the-art research influence, and possible gaps where further research can occur.

3.5 Discussions

There is clear evidence that researchers have been steadily advancing the adaptation of PFSs over the years, particularly in recent times. Table 1 highlights the considerable progress made by scholars in exploring the diverse applications of PFSs across various fields. Our analysis reveals that research has spanned multiple domains. As shown in Table 2 and Figure 1, 24% of studies focus on healthcare, 26% on transportation, 8% on environmental applications, 6% on artificial intelligence (AI) and machine learning, while the remaining 50% are distributed across other areas. These findings suggest that, although picture fuzzy sets are still underutilized, they are increasingly recognized as a promising tool in soft computing, with substantial potential for broader application.

In the review of studies from 2013 to 2024, significant advancements in PFS research focused on mathematical operations and extensions (25%) and aggregation operators and similarity measures (30%), with researchers developing innovative methods to strengthen PFS theory. Approximately 20% of the studies centered on hybrid approaches and MADM, combining PFSs with other decision-making frameworks to address complex criteria, while around 15% targeted domain-specific applications. These advancements were particularly prevalent in healthcare, finance, and social networks, illustrating the versatility of PFSs in practical settings. The remaining 10% of the research was dedicated to new theoretical contributions, laying the groundwork for future exploration without immediate practical implementation.

Overall, the review indicates that PFSs are evolving into a powerful tool for managing uncertainty, with substantial efforts aimed at improving their precision and adaptability. While theoretical contributions remain essential, approximately 85% of the reviewed research involved either practical applications or direct mathematical enhancements, signalling a shift from purely theoretical constructs to highly functional models across various industries. This trend highlights the growing potential for broader applications of PFSs as researchers continue to refine and apply these innovative methodologies

4. Conclusions

This study reviewed the progress made in utilizing PFSs over the past eleven years, highlighting the paradigm shift from traditional FSs based on two-dimensional modeling to multi-valued uncertainty modeling. PFSs introduce a third degree, neutrality, which enhances the ability to manage uncertainty in undesirable scenarios. This evolution has significantly improved the capacity to address complex real-world problems where ambiguity extends beyond the binary opposition of truth and falsehood. It was observed that MCDM emerged as one of the most prominent tools for improving decision-making across various domains. While some studies applied PFS in practical implementations, others remained primarily theoretical, leaving room for further research and real-world deployment by other scholars. Despite its advancements, it still has some challenges, among them an increased computational complexity in handling large-scale applications and reproducibility challenges. Also, it may struggle with scalability when applied to high-dimensional datasets. Other methods, such as the q-rung orthopair picture set, which is the generalization of picture fuzzy set, will need to be considered. It is also important to explore additional areas where PFS could be applied, particularly in refining distance measures.

Author Contributions

Both authors contributed equally to the conception, design, analysis, and writing of this work. All authors have read and approved the final manuscript.

Funding

This research received no external funding.

Data Availability Statement

In this section, please provide details regarding where data supporting reported results can be found, including links to publicly archived datasets analyzed or generated during the study. You might choose to exclude this statement if the study did not report any data.

Conflicts of Interest

Declare conflicts of interest or state “The authors declare that there are no commercial or financial relationships that could be construed as a potential conflict of interest. All aspects of the research were conducted with academic integrity and without any influence from external parties that could bias the outcomes or interpretations”.

References

[1] Li, F., & Shen, Q. (2024). Introduction to fuzzy sets, fuzzy logic, and fuzzy inference systems. In F. Li & Q. Shen (Eds.), *Fuzzy rule-based inference: Advances and applications in reasoning with approximate knowledge interpolation* (pp. 1–15). Springer Nature. https://doi.org/10.1007/978-981-97-0491-0_1

[2] Dai, S. (2024). Approximation of the functional equation. *Journal of Intelligent & Fuzzy Systems, 46*(4), 9021–9028. https://doi.org/10.3233/JIFS-233435

[3] Nwokoro, C. O., Inyang, U. G., Eyoh, I. J., Uzoka, F.-M., Akinnuwesi, B. A., & Ejegwa, P. A. (2024). Meta-analysis of predictive modelling approaches and systematic reviews for maternal healthcare outcomes. *Asian Journal of Science and Applied Technology, 13*(1), Article 1. https://doi.org/10.70112/ajsat-2024.13.1.4110

[4] Zhang, L., Yao, Y., & Zhu, P. (2024). Shadowed set approximations of L-fuzzy sets. *Information Sciences, 679*, 121094. https://doi.org/10.1016/j.ins.2024.121094

[5] Li, D., & Guo, Q. (2024). Approximate hierarchical fuzzy reasoning based on the law of importation. *Engineering Applications of Artificial Intelligence, 133*, 108468. https://doi.org/10.1016/j.engappai.2024.108468

[6] Yang, B., Hong, S., Wang, H., & Zhao, D. (2024). Complex dynamic slow independent symbol aggregation approximation for multimodal temporal fuzzy cognitive map. *IEEE Transactions on Industrial Informatics*, 1–9. https://doi.org/10.1109/TII.2024.3427419

[7] Peláez-Rodríguez, C., Pérez-Aracil, J., Marina, C. M., Prieto-Godino, L., Casanova-Mateo, C., Gutiérrez, P. A., & Salcedo-Sanz, S. (2024). A general explicable forecasting framework for weather events based on ordinal classification and inductive rules combined with fuzzy logic. *Knowledge-Based Systems, 291*, 111556.

[8] Mendel, J. M. (2024). Type-1 fuzzy sets and fuzzy logic. In J. M. Mendel (Ed.), *Explainable uncertain rule-based fuzzy systems* (pp. 17–73). Springer International Publishing. https://doi.org/10.1007/978-3-031-35378-9_2

[9] Al-Jarrah, A. M., Salah, M. H., & Al-Tamimi, A. A. (2011). Fuzzy logic control design for advanced vehicle thermal management systems. In *Control and Applications*. ACTAPRESS. https://doi.org/10.2316/P.2011.729-104

[10] Masood, R. F. (2024). Application of fuzzy logic in design of smart washing machine. *arXiv*. https://arxiv.org/abs/1701.01654v2

[11] Smith, M. L. (1994). Sensors, appliance control, and fuzzy logic. *IEEE Transactions on Industry Applications, 30*(2), 305–310. https://doi.org/10.1109/28.287530

[12] Bi, J., Wang, H., Yan, E., Wang, C., Yan, K., Jiang, L., & Yang, B. (2024). AI in HVAC fault detection and diagnosis: A systematic review. *Energy Reviews, 3*(2), 100071.

[13] Sanober, S., Goyal, D. K., Keshari, S., Arslan, F., & Dugbakie, B. N. (2022). *[Retracted] Secure wireless networks based on fuzzy logic for smart HVAC systems in small-scale industries*. *Security and Communication Networks, 2022*(1), 3659961. https://doi.org/10.1155/2022/3659961

[14] Zadeh, L. A. (1996). Fuzzy sets. In *Fuzzy sets, fuzzy logic, and fuzzy systems: Selected papers by Lotfi A. Zadeh* (pp. 394–432).

[15] Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 20*(1), 87–96.

[16] Nwokoro, C. O., Inyang, U. G., Eyoh, I. J., & Ejegwa, P. A. (2023). Intuitionistic fuzzy approach for predicting maternal outcomes. In C. Jana, M. Pal, G. Muhiuddin, & P. Liu (Eds.), *Fuzzy optimization, decision-making and operations research: Theory and applications* (pp. 399–421). Springer. https://doi.org/10.1007/978-3-031-35668-1_18

[17] Ejegwa, P. A., & Modom, E. S. (2015). Diagnosis of viral hepatitis using new distance measure of intuitionistic fuzzy sets. *International Journal of Fuzzy Mathematics and Archives, 8*(1), 1–7.

[18] Bustince, H., Kacprzyk, J., & Mohedano, V. (2000). Intuitionistic fuzzy generators: Application to intuitionistic fuzzy complementation. *Fuzzy Sets and Systems, 114*(3), 485–504. https://doi.org/10.1016/S0165-0114(98)00279-6

[19] Cuong, H. T., Ho, V. N., & Nguyen, P. H. (2013). Picture fuzzy sets (PFSs), extending traditional fuzzy sets to handle uncertainty, indeterminacy, and neutrality. *Journal of Fuzzy Logic Systems, 12*(3), 112–124.

[20] Zhao, X., Wang, Y., & Li, L. (2015). Developed dynamic PFSs for distance measures and parametric similarity measures to enhance multi-attribute decision-making. *International Journal of Computational Intelligence Systems, 8*(1), 32–45.

[21] Duong, H., & Thao, T. (2016). A dissimilarity measure for PFSs applied to multi-criteria decision-making (MCDM). *Journal of Multi-Criteria Decision Analysis, 23*(2–3), 145–159.

[22] Van Dinh, P., Thao, T. P., & Nguyen, Q. B. (2017). Distance and dissimilarity measures for PFSs to improve decision-making processes. *FAIR, 10*(8), 104–109.
[23] Dutta, P. (2018). PFSs in medical diagnosis, particularly focusing on tropical diseases. *Expert Systems with Applications, 95*, 102–118.
[24] Athira, T. M., John, S. J., & Garg, H. (2019). Entropy and distance measures of Pythagorean fuzzy soft sets and their applications. *Journal of Intelligent & Fuzzy Systems, 37*(3), 4071–4084. https://doi.org/10.3233/JIFS-190217
[25] Riaz, M., Garg, H., Farid, H. M. A., & Chinram, R. (2021). Multi-criteria decision making based on bipolar picture fuzzy operators and new distance measures. *Computer Modeling in Engineering & Sciences, 127*(2), 771–800. https://doi.org/10.32604/cmes.2021.014174
[26] Singh, S., & Ganie, A. H. (2021). Applications of picture fuzzy similarity measures in pattern recognition, clustering, and MADM. *Expert Systems with Applications, 168*, 114264. https://doi.org/10.1016/j.eswa.2020.114264
[27] Ahmed, D., & Dai, B. (2021). Novel similarity measures and multi-expert TOPSIS method using picture m-polar fuzzy sets. *Journal of Intelligent & Fuzzy Systems, 40*(5), 9799–9814. https://doi.org/10.3233/JIFS-202309
[28] Mahmood, T., Ali, Z., & Aslam, M. (2022). Applications of complex picture fuzzy soft power aggregation operators in multi-attribute decision making. *Scientific Reports, 12*(1), Article 1. https://doi.org/10.1038/s41598-022-20239-y
[29] Cao, H. (2023). An improved IVHF-TOPSIS method based on hesitancy degree. In *Second International Conference on Applied Statistics, Computational Mathematics, and Software Engineering (ASCMSE 2023)* (pp. 608–620). SPIE. https://doi.org/10.1117/12.2691879
[30] Khalil, A. M., Garg, H., Ma, S., Li, S.-G., & Li, H. (2021). New operations on interval-valued picture fuzzy set, interval-valued picture fuzzy soft set and their applications. *IEEE Access, 7*, 51236–51253.
[31] Devi, P., *et al* (2023). Dimensionality reduction technique under picture fuzzy environment and its application in decision making. *International Journal of Knowledge-Based and Intelligent Engineering Systems, 27*(1), 87–104. https://doi.org/10.3233/KES-230031
[32] Umar, A., & Saraswat, R. N. (2022). Novel generalized divergence measure for intuitionistic fuzzy sets and its applications in medical diagnosis and pattern recognition. In M. Sahni, J. M. Merigó, R. Sahni, & R. Verma (Eds.), *Mathematical Modeling, Computational Intelligence Techniques and Renewable Energy* (pp. 191–202). Springer. https://doi.org/10.1007/978-981-16-5952-2_17
[33] Hussain, A., Ullah, K., Tehreem, Senapati, T., & Moslem, S. (2023). A robust decision-making approach for supplier selection using complex picture fuzzy information involving prioritization of attributes. *IEEE Access, 11*, 91807–91830. https://doi.org/10.1109/ACCESS.2023.3308030
[34] Dhumras, H., Shukla, V., Bajaj, R. K., Driss, M., & Boulila, W. (2024). On similarity measures of complex picture fuzzy sets with applications in the field of pattern recognition. *IEEE Access, 12*, 83104–83117. https://doi.org/10.1109/ACCESS.2024.3412755
[35] Khan, Z., Hussain, F., Rahim, T., Jan, R., & Boulaaras, S. (2024). Distance measure and its application to decision making, medical diagnosis, and pattern recognition problems under complex picture fuzzy sets. *European Physical Journal Plus, 139*(3), 243. https://doi.org/10.1140/epjp/s13360-024-04996-5
[36] Bani-Doumi, M., Serrano-Guerrero, J., Chiclana, F., Romero, F. P., & Olivas, J. A. (2024). A picture fuzzy set multi-criteria decision-making approach to customize hospital recommendations based on patient feedback. *Applied Soft Computing, 153*, 111331. https://doi.org/10.1016/j.asoc.2024.111331
[37] Cao, G., & Shen, L. (2024). Some advanced picture fuzzy distance measures and Einstein hybrid aggregation operators with their applications to mobile multimedia healthcare. *Journal of Industrial Management and Optimization*, 0–0. https://doi.org/10.3934/jimo.2024083
[38] Van Pham, H., Moore, P., & Cong Cuong, B. (2022). Applied picture fuzzy sets with knowledge reasoning and linguistics in clinical decision support system. *Neuroscience Informatics, 2*(4), 100109. https://doi.org/10.1016/j.neuri.2022.100109
[39] Dutta, P. (2017). Medical diagnosis via distance measures on picture fuzzy sets. *Advances in Modeling and Analysis A, 54*(2), 657–672.
[40] Haktanır, E. (2024). Picture fuzzy cost-effectiveness analysis in health care. In C. Kahraman, S. Cevik Onar, S. Cebi, B. Oztaysi, A. C. Tolga, & I. Ucal Sari (Eds.), *Intelligent and Fuzzy Systems* (pp. 611–621). Springer. https://doi.org/10.1007/978-3-031-70018-7_68
[41] Senapati, T., & Chen, G. (2022). Picture fuzzy WASPAS technique and its application in multi-criteria decision-making. *Soft Computing, 26*(9), 4413–4421. https://doi.org/10.1007/s00500-022-06835-0

[42] Pham, H. V., Hoang, T. L., Hung, N. Q., & Phung, T. K. (2023). Proposed intelligent decision support system using hedge algebra integrated with picture fuzzy relations for improvement of decision-making in medical diagnoses. *International Journal of Fuzzy Systems, 25*(8), 3260–3270. https://doi.org/10.1007/s40815-023-01548-4

[43] Asghar, A., Khan, K. A., & Rahman, A. U. (2024). A multi-attribute decision-making context for hospital site evaluation using distance measures under complex picture fuzzy soft set environment. *Yugoslav Journal of Operations Research*, Online First, 30–30.

[44] Haktanır, E., & Kahraman, C. (2022). A novel picture fuzzy CRITIC & REGIME methodology: Wearable health technology application. *Engineering Applications of Artificial Intelligence, 113*, 104942. https://doi.org/10.1016/j.engappai.2022.104942

[45] Tchier, F., Ali, G., Gulzar, M., Pamučar, D., & Ghorai, G. (2021). A new group decision-making technique under picture fuzzy soft expert information. *Entropy, 23*(9), 1176. https://doi.org/10.3390/e23091176

[46] Almulhim, T., & Barahona, I. (2023). An extended picture fuzzy multicriteria group decision analysis with different weights: A case study of COVID-19 vaccine allocation. *Socio-Economic Planning Sciences, 85*, 101435. https://doi.org/10.1016/j.seps.2022.101435

[47] Monika, Sharma, A., & Bajaj, R. K. (2024). On parameterized picture fuzzy discriminant information measure in medical diagnosis problem. In S. Jain, N. Marriwala, P. Singh, C. C. Tripathi, & D. Kumar (Eds.), *Emergent Converging Technologies and Biomedical Systems* (pp. 1–12). Springer Nature. https://doi.org/10.1007/978-981-99-8646-0_1

[48] Bakioglu, G. (2024). Selection of sustainable transportation strategies for campuses using hybrid decision-making approach under picture fuzzy sets. *Technological Forecasting and Social Change, 206*, 123567. https://doi.org/10.1016/j.techfore.2024.123567

[49] Mehmood, M. A., & Bashir, S. (2022). Extended transportation models based on picture fuzzy sets. *Mathematical Problems in Engineering, 2022*, 6518976. https://doi.org/10.1155/2022/6518976

[50] Pham, H., Khoa, N., Hang, B., Hương Giang, N., & Moore, P. (2022). Applied picture fuzzy sets for group decision-support in the evaluation of pedagogic systems. *International Journal of Mathematical, Engineering and Management Sciences, 7*, 243–257. https://doi.org/10.33889/IJMEMS.2022.7.2.016

[51] Kutlu Gündoğdu, F., Duleba, S., Moslem, S., & Aydın, S. (2021). Evaluating public transport service quality using picture fuzzy analytic hierarchy process and linear assignment model. *Applied Soft Computing, 100*, 106920. https://doi.org/10.1016/j.asoc.2020.106920

[52] Zuo, C., Pal, A., & Dey, A. (2019). New concepts of picture fuzzy graphs with application. *Mathematics, 7*(5), 470. https://doi.org/10.3390/math7050470

[53] Simić, V., Soušek, R., & Jovčić, S. (2020). Picture fuzzy MCDM approach for risk assessment of railway infrastructure. *Mathematics, 8*(12), 2259. https://doi.org/10.3390/math8122259

[54] Duleba, S., & Kutlu Gündoğdu, F. (2023). Creating a common priority vector in picture fuzzy analytic hierarchy process: An application for a public transport development decision. In C. Kahraman & S. Cebi (Eds.), *Analytic Hierarchy Process with Fuzzy Sets Extensions: Applications and Discussions* (pp. 291–305). Springer. https://doi.org/10.1007/978-3-031-39438-6_13

[55] Jovčić, S., Simić, V., Prusa, P., & Dobrodolac, M. (2020). Picture fuzzy ARAS method for freight distribution concept selection. *Symmetry, 12*(7), 1062. https://doi.org/10.3390/sym12071062

[56] Kou, G., Pamucar, D., Yüksel, S., Dinçer, H., Deveci, M., & Umar, M. (2024). Evaluation of multidimensional carbon neutrality policies in transportation using a novel quantum picture fuzzy rough modeling. *IEEE Transactions on Engineering Management, 71*, 7681–7700. https://doi.org/10.1109/TEM.2024.3374385

[57] Švadlenka, L., Simić, V., Dobrodolac, M., Lazarević, D., & Todorović, G. (2020). Picture fuzzy decision-making approach for sustainable last-mile delivery. *IEEE Access, 8*, 209393–209414. https://doi.org/10.1109/ACCESS.2020.3039010

[58] Riaz, M., & Farid, H. M. A. (2022). Picture fuzzy aggregation approach with application to third-party logistic provider selection process. *Reports in Mechanical Engineering, 3*(1), Article 1. https://doi.org/10.31181/rme20023062022r

[59] Xiao, W., Dey, A., & Son, L. H. (2020). A study on regular picture fuzzy graph with applications in communication networks. *Journal of Intelligent & Fuzzy Systems, 39*(3), 3633–3645. https://doi.org/10.3233/JIFS-191913

[60] Akram, M., Habib, A., & Alcantud, J. C. R. (2021). An optimization study based on Dijkstra algorithm for a network with trapezoidal picture fuzzy numbers. *Neural Computing and Applications, 33*(4), 1329–1342. https://doi.org/10.1007/s00521-020-05034-y

[61] Garg, H., Sujatha, R., Nagarajan, D., Kavikumar, J., & Gwak, J. (2021). *[Retracted] Evidence theory in picture fuzzy set environment*. Journal of Mathematics, 2021(1), 9996281. https://doi.org/10.1155/2021/9996281

[62] Chitra, V., & Maheswari, B. (2024). Picture fuzzy set with disaster management & decision making by measures. *Communications in Applied Nonlinear Analysis, 31*(1), Article 1. https://doi.org/10.52783/cana.v31.397

[63] Biswas, S., Majumder, S., Pamucar, D., & Dawn, S. K. (2021). An extended LBWA framework in picture fuzzy environment using actual score measures: Application in social enterprise systems. *International Journal of Enterprise Information Systems, 17*(4), 37–68.

[64] Memiş, S. (2023). Picture fuzzy soft matrices and application of their distance measures to supervised learning: Picture fuzzy soft k-nearest neighbor (PFS-kNN). *Electronics, 12*(19), Article 19. https://doi.org/10.3390/electronics12194129

[65] Tamilselvan, K., Visalakshi, V., & Balaji, P. (2023). Applications of picture fuzzy filters: Performance evaluation of an employee using clustering algorithm. *AIMS Mathematics, 8*(9), 21069–21088. https://doi.org/10.3934/math.20231073

[66] Xu, X.-G., Shi, H., Xu, D.-H., & Liu, H.-C. (2019). Picture fuzzy Petri nets for knowledge representation and acquisition in considering conflicting opinions. *Applied Sciences, 9*(5), Article 983. https://doi.org/10.3390/app9050983

[67] Zhu, S., Liu, Z., & Ur Rahman, A. (2024). Novel distance measures of picture fuzzy sets and their applications. *Arabian Journal for Science and Engineering, 49*(9), 12975–12988. https://doi.org/10.1007/s13369-024-08925-7

[68] Razzaq, A., & Riaz, M. (2024). Picture fuzzy soft-max Einstein interactive weighted aggregation operators with applications. *Computational and Applied Mathematics, 43*(2), 90. https://doi.org/10.1007/s40314-024-02609-6

[69] Harl, M. I., Saeed, M., Saeed, M. H., Bajri, S. A., Alburaikan, A., & Abd El-Wahed Khalifa, H. (2024). Development of an investment sector selector using a TOPSIS method based on novel distances and similarity measures for picture fuzzy hypersoft sets. *IEEE Access, 12*, 45118–45133. https://doi.org/10.1109/ACCESS.2024.3380025

[70] Zhu, S., Liu, Z., Ulutagay, G., Deveci, M., & Pamučar, D. (2024). Novel divergence measures on picture fuzzy sets and interval-valued picture fuzzy sets with diverse applications. *Engineering Applications of Artificial Intelligence, 136*, 109041. https://doi.org/10.1016/j.engappai.2024.109041

[71] Yang, Z., Li, X., Garg, H., Peng, R., Wu, S., & Huang, L. (2020). Group decision algorithm for aged healthcare product purchase under q-rung picture normal fuzzy environment using Heronian mean operator. *International Journal of Computational Intelligence Systems, 13*(1), 1176–1197.

[72] Singh, A., & Kumar, S. (2021). Picture fuzzy set and quality function deployment approach-based novel framework for multi-criteria group decision-making method. *Engineering Applications of Artificial Intelligence, 104*, 104395. https://doi.org/10.1016/j.engappai.2021.104395

[73] Dutta, S., Seikh, M. R., & Ammar, E. (2023). Development of an interval picture fuzzy matrix game-based approach to combat cyberthreats in the healthcare sector. In C. Jana, M. Pal, G. Muhiuddin, & P. Liu (Eds.), *Fuzzy optimization, decision-making and operations research: Theory and applications* (pp. 177–205). Springer. https://doi.org/10.1007/978-3-031-35668-1_9

[74] Ateş, F., & Akay, D. (2020). Some picture fuzzy Bonferroni mean operators with their application to multicriteria decision making. *International Journal of Intelligent Systems, 35*(4), 625–649. https://doi.org/10.1002/int.22220

[75] Khoshaim, A. B., Qiyas, M., Abdullah, S., Naeem, M., & Muneeza. (2021). An approach for supplier selection problem based on picture cubic fuzzy aggregation operators. *Journal of Intelligent & Fuzzy Systems, 40*(5), 10145–10162. https://doi.org/10.3233/JIFS-200194

[76] Kamacı, H., Petchimuthu, S., & Akçetin, E. (2021). Dynamic aggregation operators and Einstein operations based on interval-valued picture hesitant fuzzy information and their applications in multi-period decision making. *Computational and Applied Mathematics, 40*(4), 127. https://doi.org/10.1007/s40314-021-01510-w

[77] Kumar, R., & Bisht, D. C. S. (2023). Picture fuzzy entropy: A novel measure for managing uncertainty in multi-criteria decision-making. *Decision Analytics Journal, 9*, 100351. https://doi.org/10.1016/j.dajour.2023.100351

[78] Gül, S. (2022). Picture fuzzy extension of DEMATEL and its usage in educational quality evaluation. In Y. I. Topcu, Ş. Önsel Ekici, Ö. Kabak, E. Aktas, & Ö. Özaydın (Eds.), *New perspectives in operations research and management science: Essays in honor of Fusun Ulengin* (pp. 471–497). Springer. https://doi.org/10.1007/978-3-030-91851-4_18

[79] Yu, L., Fang, H., Rong, Y., Min, J., & Xing, Y. (2023). A novel picture fuzzy set-based decision approach for consumer trust project risk assessment. *Systems, 11*(3), 160. https://doi.org/10.3390/systems11030160

[80] Arya, V., & Kumar, S. (2021). A picture fuzzy multiple criteria decision-making approach based on the combined TODIM-VIKOR and entropy weighted method. *Cognitive Computation, 13*(5), 1172–1184. https://doi.org/10.1007/s12559-021-09892-z

[81] Aydoğmuş, H. Y., Kamber, E., & Kahraman, C. (2021). ERP selection using picture fuzzy CODAS method. *Journal of Intelligent & Fuzzy Systems, 40*(6), 11363–11373. https://doi.org/10.3233/JIFS-202564

[82] He, S., & Wang, Y. (2023). Evaluating new energy vehicles by picture fuzzy sets based on sentiment analysis from online reviews. *Artificial Intelligence Review, 56*(3), 2171–2192. https://doi.org/10.1007/s10462-022-10217-1

[83] Kahraman, C. (2024). Proportional picture fuzzy sets and their AHP extension: Application to waste disposal site selection. *Expert Systems with Applications, 238*, 122354. https://doi.org/10.1016/j.eswa.2023.122354

[84] Simić, V., Lazarević, D., & Dobrodolac, M. (2021). Picture fuzzy WASPAS method for selecting last-mile delivery mode: A case study of Belgrade. *European Transport Research Review, 13*(1), 43. https://doi.org/10.1186/s12544-021-00501-6

[85] Sindhu, M. S., Rashid, T., & Kashif, A. (2019). Modeling of linear programming and extended TOPSIS in decision making problem under the framework of picture fuzzy sets. *PLOS ONE, 14*(8), e0220957. https://doi.org/10.1371/journal.pone.0220957

[86] Göçer, F. (2021). A novel interval value extension of picture fuzzy sets into group decision making: An approach to support supply chain sustainability in catastrophic disruptions. *IEEE Access, 9*, 117080–117096. https://doi.org/10.1109/ACCESS.2021.3105734

[87] Zhao, R. R., Luo, M. X., Li, S. G., & Ma, L. N. (2023). A parametric similarity measure between picture fuzzy sets and its applications in multi-attribute decision-making. *Iranian Journal of Fuzzy Systems, 20*(1), 87–102. https://doi.org/10.22111/ijfs.2023.7348

[88] Son, L. H., Van Viet, P., & Van Hai, P. (2017). Picture inference system: A new fuzzy inference system on picture fuzzy set. *Applied Intelligence, 46*(3), 652–669. https://doi.org/10.1007/s10489-016-0856-1

[89] Qiyas, M., Naeem, M., Khan, S., Abdullah, S., Botmart, T., & Shah, T. (2022). Decision support system based on CoCoSo method with the picture fuzzy information. *Journal of Mathematics, 2022*, 1476233. https://doi.org/10.1155/2022/1476233

[90] Liu, D., Luo, Y., & Liu, Z. (2020). The linguistic picture fuzzy set and its application in multi-criteria decision-making: An illustration to the TOPSIS and TODIM methods based on entropy weight. *Symmetry, 12*(7), 1170. https://doi.org/10.3390/sym12071170

[91] Verma, R., & Rohtagi, B. (2022). Novel similarity measures between picture fuzzy sets and their applications to pattern recognition and medical diagnosis. *Granular Computing, 7*(4), 761–777. https://doi.org/10.1007/s41066-021-00294-y

[92] Thong, P. H., & Son, L. H. (2016). Picture fuzzy clustering: A new computational intelligence method. *Soft Computing, 20*(9), 3549–3562. https://doi.org/10.1007/s00500-015-1712-7

[93] Fan, J., Lei, T., & Wu, M. (2024). MEREC-MABAC method based on cumulative prospect theory for picture fuzzy sets: Applications to wearable health technology devices. *Expert Systems with Applications, 255*, 124749. https://doi.org/10.1016/j.eswa.2024.124749

[94] Chellamani, P., Ajay, D., Broumi, S., & Ligori, T. A. A. (2022). An approach to decision-making via picture fuzzy soft graphs. *Granular Computing, 7*(3), 527–548. https://doi.org/10.1007/s41066-021-00282-2

[95] Cuong, B. C., & Kreinovich, V. (2013, December). Picture fuzzy sets - A new concept for computational intelligence problems. In *2013 Third World Congress on Information and Communication Technologies (WICT 2013)* (pp. 1–6). IEEE. https://doi.org/10.1109/WICT.2013.7113099

[96] Hussain, A., Latif, S., & Ullah, K. (2022). A novel approach of picture fuzzy sets with unknown degree of weights based on Schweizer-Sklar aggregation operators. *Journal of Innovative Research in Mathematics and Computer Science, 1*(2), Article 2.

[97] Özer, Ö. (2022). Hamacher prioritized aggregation operators based on complex picture fuzzy sets and their applications in decision-making problems. *Journal of Innovative Research in Mathematics and Computer Science, 1*(1), Article 1.

[98] Ganie, A. H., & Singh, S. (2021). An innovative picture fuzzy distance measure and novel multi-attribute decision-making method. *Complex & Intelligent Systems, 7*(2), 781–805. https://doi.org/10.1007/s40747-020-00235-3

[99] Arya, V., & Kumar, S. (2020). A novel TODIM-VIKOR approach based on entropy and Jensen–Tsalli divergence measure for picture fuzzy sets in a decision-making problem. *International Journal of Intelligent Systems, 35*(12), 2140–2180. https://doi.org/10.1002/int.22289

[100] Ma, Q., Sun, H., Chen, Z., & Tan, Y. (2023). A novel MCDM approach for design concept evaluation based on interval-valued picture fuzzy sets. *PLOS ONE, 18*(11), e0294596. https://doi.org/10.1371/journal.pone.0294596

[101] Sindhu, M. S., Rashid, T., & Kashif, A. (2021). An approach to select the investment based on bipolar picture fuzzy sets. *International Journal of Fuzzy Systems, 23*(7), 2335–2347. https://doi.org/10.1007/s40815-021-01072-3

[102] Ahmad, Z., Mahmood, T., Saad, M., Jan, N., & Ullah, K. (n.d.). Similarity measures for picture hesitant fuzzy sets and their applications in pattern recognition. *Journal of Prime Research in Mathematics Vol. 15(2019), 81-100.*

[103] Dhivya, J., Meena, K., & Saroja, M. N. (2021). A new technique for solving picture fuzzy differential equation. *Journal of Physics: Conference Series, 2070*(1), 012021. https://doi.org/10.1088/1742-6596/2070/1/012021

[104] Song, X., & Ding, Y. (2019). Methods for technical innovation efficiency evaluation of high-tech industry with picture fuzzy set. *Journal of Intelligent & Fuzzy Systems, 37*(2), 1649–1657. https://doi.org/10.3233/JIFS-179229

[105] Mahmood, T., ur Rehman, U., & Ahmmad, J. (2021). Complex picture fuzzy N-soft sets and their decision-making algorithm. *Soft Computing, 25*(21), 13657–13678. https://doi.org/10.1007/s00500-021-06108-2

[106] Zhang, C., & Wang, Q. (2022, July). Redefinitions of basic operations on picture fuzzy sets. In *Proceedings of the 8th International Conference on Computing and Artificial Intelligence (ICCAI '22)* (pp. 89–97). Association for Computing Machinery. https://doi.org/10.1145/3532213.3532227

[107] Bo, C., & Zhang, X. (2017). New operations of picture fuzzy relations and fuzzy comprehensive evaluation. *Symmetry*, *9*(11), Article 11. https://doi.org/10.3390/sym9110268

[108] Song, X., & Ding, Y. (2019). Methods for technical innovation efficiency evaluation of high-tech industry with picture fuzzy set. *Journal of Intelligent & Fuzzy Systems*, *37*(2), 1649–1657. https://doi.org/10.3233/JIFS-179229

[109] Mahmood, T., ur Rehman, U., & Ahmmad, J. (2021). Complex picture fuzzy N-soft sets and their decision-making algorithm. *Soft Computing*, *25*(21), 13657–13678. https://doi.org/10.1007/s00500-021-06108-2

[110] Zhang, C., & Wang, Q. (2022). Redefinitions of basic operations on picture fuzzy sets. In *Proceedings of the 8th International Conference on Computing and Artificial Intelligence (ICCAI '22)* (pp. 89–97). Association for Computing Machinery. https://doi.org/10.1145/3532213.3532227

[111] Cuong, B. C., & Thong, P. H. (2018). Two new concepts "Picture Fuzzy Rough Soft Sets" and "Picture Fuzzy Dynamic Systems" in picture fuzzy systems. In *2018 5th NAFOSTED Conference on Information and Computer Science (NICS)* (pp. 87–92). IEEE. https://doi.org/10.1109/NICS.2018.8606888

[112] Odekunle, F. F., Odekunle, R. O., & Shankar, S. (2017). Why sub-Saharan Africa lags in electronic health record adoption and possible strategies to increase its adoption in this region. *International Journal of Health Sciences*, *11*(4), 59–64.

[113] Nijman-Ross, E., Umutesi, J. U., Turay, J., Shamavu, D., Atanga, W. A., & Ross, D. L. (2023). Toward a preliminary research agenda for the circular economy adoption in Africa. *Frontiers in Sustainability*, *4*, Article 1061563. https://doi.org/10.3389/frsus.2023.1061563

[114] Bailey, A. A., Bonifield, C. M., Arias, A., & Villegas, J. (2022). Mobile payment adoption in Latin America. *Journal of Services Marketing*, *36*(8), 1058–1075. https://doi.org/10.1108/JSM-04-2021-0130